Solutions - Midterm Exam

(October 16th @ 5:30 pm)

Presentation and clarity are very important! Show your procedure!

PROBLEM 1 (20 PTS)

a) Complete the following table. The decimal numbers are unsigned: (5 pts.)

Decimal	BCD	Binary	Reflective Gray Code
33	00110011	100001	110001
57	01010111	111001	100101
133	000100110011	10000101	11000111

b) Complete the following table. The decimal numbers are signed. Use the fewest number of bits in each case: (12 pts.)

	REPRESENTATION		
Decimal	Sign-and-magnitude	1's complement	2's complement
-26	111010	100101	100110
-64	11000000	10111111	1000000
23	010111	010111	010111
-33	1100001	1011110	1011111
-4	1100	1011	100
-9	11001	10110	10111

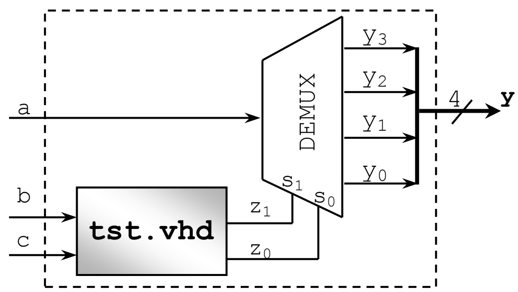
c) Convert the following decimal numbers to their 2's complement representations. (3 pts)

- ✓ -21.375
  21.375 = 010101.011 ⇒ -21.375 = 101010.101
- ✓ 19.125
  +19.125 = 010011.001

PROBLEM 2 (16 PTS)

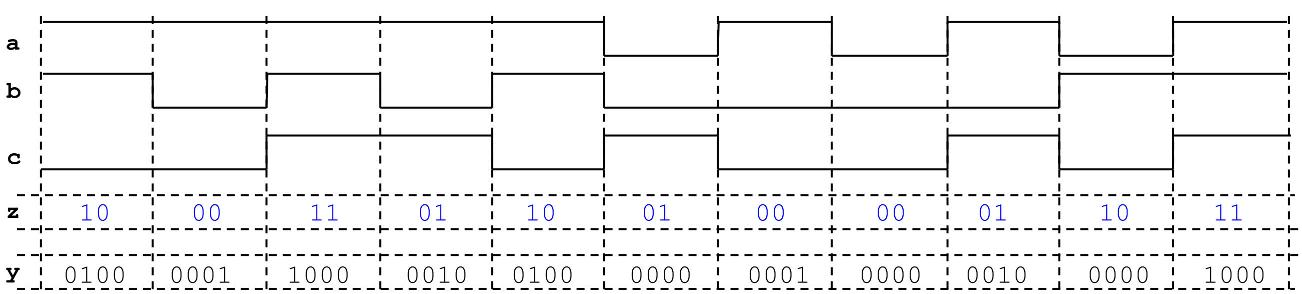
- Complete the timing diagram of the following circuit. The VHDL code (tst.vhd) corresponds to the shaded circuit.

$z = z_1 z_0$, $y = y_3 y_2 y_1 y_0$




```
library ieee;
use ieee.std_logic_1164.all;

entity tst is
  port (b,c : in std_logic;
        z: out std_logic_vector(1 downto 0));
end tst;
```

```
architecture bhv of tst is

begin

  process (b, c)
  begin
     z <= b & c;
     if c = '1' then
        z <= b&'1';
     end if;
  end process;

end bhv;
```



- Get the Boolean equations for y_3, y_2, y_1, y_0 based on a, b, c (4 pts)

$z_1 = b$, $z_0 = c$

$y_3(a,b,c) = z_1 z_0 . a = bca$

$y_2(a,b,c) = z_1 \bar{z}_0 . a = b\bar{c}a$

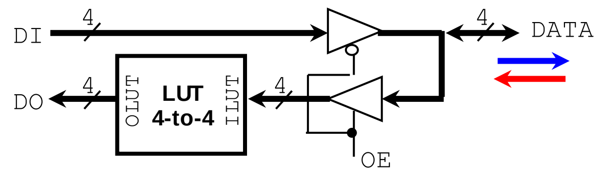
$y_1(a,b,c) = \bar{z}_1 z_0 . a = \bar{b}ca$

$y_0(a,b,c) = \bar{z}_1 \bar{z}_0 . a = \bar{b}\bar{c}a$

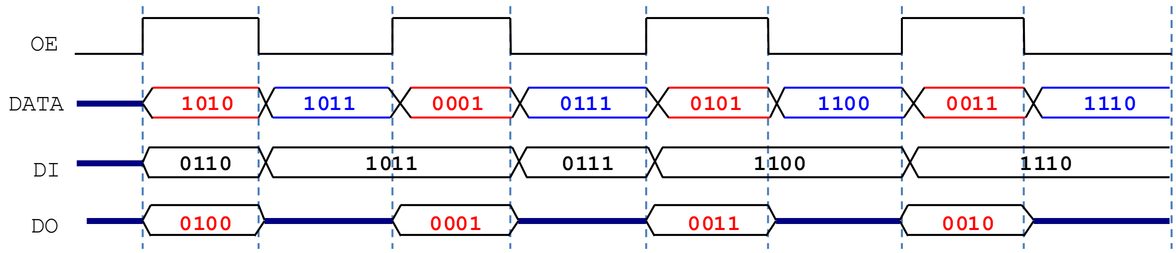
PROBLEM 3 (10 PTS)

- Given the following circuit, complete the timing diagram (signals DO and $DATA$).
  The LUT 4-to-4 implements the following function: $OLUT = \lceil sqrt(ILUT) \rceil$. For example: $ILUT = 1100 \rightarrow OLUT = 0100$

Input data to LUT is treated as an unsigned number.




```
ceil(sqrt(10))= 4
ceil(sqrt(1)) = 1
ceil(sqrt(5)) = 3
ceil(sqrt(3)) = 2
```



PROBLEM 4 (11 PTS)

- The figure below depicts the entire memory space of a microprocessor. Each memory address occupies one byte. 1KB = 2^{10} bytes, 1MB = 2^{20} bytes, 1GB = 2^{30} bytes
  - ✓ What is the size (in bytes, KB, or MB) of the memory space? What is the address bus size of the microprocessor? (2 pts.)

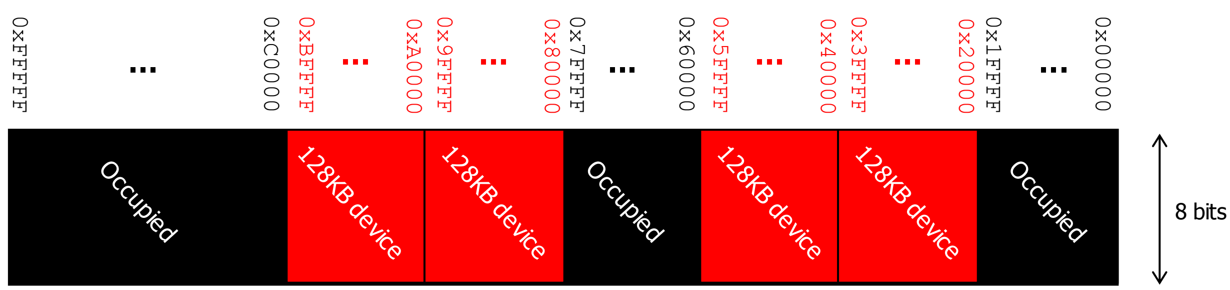
    Address space: `0x00000` to `0xFFFFF`. To represent all these addresses, we require 20 bits. So, the address bus size of the microprocessor is 20 bits. The size of the memory space is then 2^{20} = 1 MB.

  - ✓ If we have a memory chip of 128 KB, how many bits do we require to address those 128 KB of memory? (1 pt.)

    128 KB memory device: 128KB = 2^{17} bytes. Thus, we require 17 bits to address the memory device.

  - ✓ We want to connect the 128 KB memory chip to the microprocessor. For optimal implementation, we must place those 128 KB in an address range where every address shares some MSBs. Provide a list of all the possible address ranges that the 128 KB memory chip can occupy. You can only use the non-occupied portions of the memory space as shown below.

    ▫ 0x20000 to 0x3FFFF ▫ 0x40000 to 0x5FFFF ▫ 0x80000 to 0x9FFFF ▫ 0xA0000 to 0xBFFFF



PROBLEM 5 (17 PTS)

a) Perform the following additions and subtractions of the following unsigned integers. Use the fewest number of bits n to represent both operators. Indicate every carry (or borrow) from c_0 to c_n (or b_0 to b_n). For the addition, determine whether there is an overflow. For the subtraction, determine whether we need to keep borrowing from a higher byte. (6 pts)

✓ 29 - 50

```
Borrow out! →  b6=1 b5=0 b4=0 b3=0 b2=1 b1=0 b0=0

   29 = 0x1D =       0  1  1  1  0  1  -
   50 = 0x32 =       1  1  0  0  1  0
   ----------------------------------
                     1  0  1  0  1  1
```

✓ 42 + 36

```
                   c6=1 c5=0 c4=0 c3=0 c2=0 c1=0 c0=0

   42 = 0x2A =       1  0  1  0  1  0  +
   36 = 0x24 =       1  0  0  1  0  0
   ----------------------------------
Overflow! →     1    0  0  1  1  1  0
```

b) Perform the following operations, where numbers are represented in 2's complement. Indicate every carry from c_0 to c_n. For each case, use the fewest number of bits to represent the summands and the result so that overflow is avoided. (8 pts)

✓ -79 + 62

```
n = 8 bits          c8=0 c7=0 c6=1 c5=1 c4=0 c3=0 c2=0 c1=0 c0=0

c8⊕c7=0          62 = 0 0 1 1 1 1 1 0 +
No Overflow     -79 = 1 0 1 1 0 0 0 1
                -------------------------
                -17 = 1 1 1 0 1 1 1 1

-62 + 79 = -17 ∈ [-2^7, 2^7-1] → no overflow
```

✓ -26 - 52

```
n = 7 bits          c7=1 c6=0 c5=0 c4=1 c3=1 c2=0 c1=0 c0=0

c7⊕c6=1        -52 = 1 0 0 1 1 0 0 +
Overflow!      -26 = 1 1 0 0 1 1 0
               ----------------------
                     0 1 1 0 0 1 0

-52 -26 = -78 ∉ [-2^6, 2^6-1] → overflow!
```

To avoid overflow: n = 8 bits (sign-extension)

```
c8⊕c7=0
No Overflow     c8=1 c7=1 c6=0 c5=0 c4=1 c3=1 c2=0 c1=0 c0=0

               -52 = 1 1 0 0 1 1 0 0 +
               -26 = 1 1 1 0 0 1 1 0
               -------------------------
                     1 0 1 1 0 0 1 0

-52 -26 = -78 ∈ [-2^7, 2^7-1] → no overflow
```

c) Perform binary multiplication of the following numbers that are represented in 2's complement arithmetic. (3 pts)

✓ 7 x -8

```
0 1 1 1 x   ➡    0 1 1 1 x
1 0 0 0          1 0 0 0
               ---------
                 0 0 0 0
               0 0 0 0
             0 0 0 0
           0 1 1 1
         ---------------
         0 1 1 1 0 0 0
               ⬇
         1 0 0 1 0 0 0
```

PROBLEM 6 (10 PTS)

- Sketch the circuit that computes $|A - B| \times 2$, where A, B are 4-bit unsigned numbers. For example: $A = 0101, B = 1101 \rightarrow |A - B| \times 2 = 8 \times 2 = 16$. You can only use full adders and logic gates. Your circuit must avoid overflow.

$A = a_3a_2a_1a_0$, $B = b_3b_2b_1b_0$

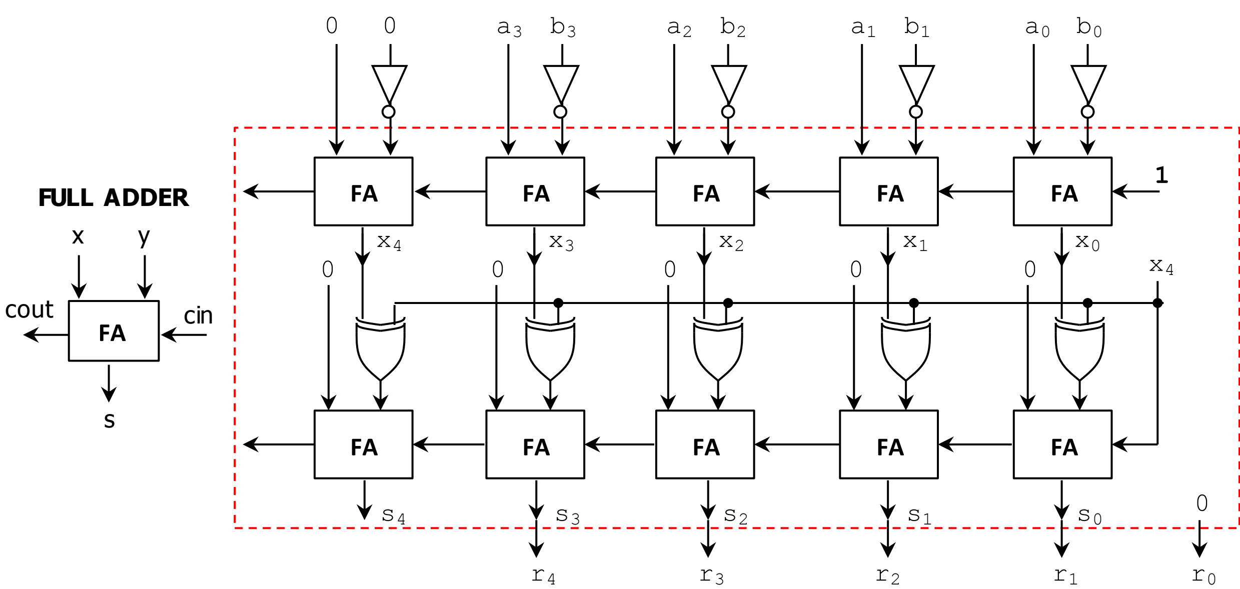
$A, B \in [0,15] \rightarrow A, B$ require 4 bits in unsigned representation. However, to get the proper result of $A - B$, we need to use the 2C representation, where A, B require 5 bits in 2C.

✓ $X = A - B \in [-15,15]$ requires 5 bits in 2C. Thus, we need to zero-extend A and B to convert them to 2C representation.

✓ $|X| = |A - B| \in [0,15]$ requires 5 bits in 2C. Thus, the second operation $0 \pm X$ only requires 5 bits.

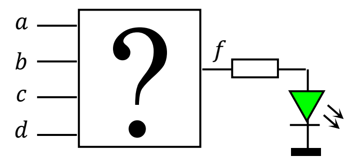
- If $x_4 = 1 \rightarrow X < 0 \rightarrow$ we do $0 - X$.
- If $x_4 = 0 \rightarrow X \geq 0 \rightarrow$ we do $0 + X$.

✓ $R = |A - B| \times 2 \in [0,30]$ requires 6 bits in 2C. Note that the MSB is always 0. The unsigned result only requires 5 bits.



PROBLEM 7 (16 PTS)

- An LED is lit ($f = 1$) when the combination of four active-high switches (a, b, c, d) represents an unsigned integer that is odd and prime, otherwise $f = 0$. For example: if $abcd = 0001 \rightarrow f = 0$. If $abcd = 1011 \rightarrow f = 1$.



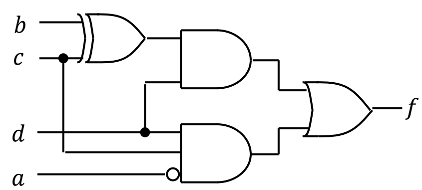
a) Provide the simplified expression for f and sketch this circuit using logic gates. (4 pts)

a	b	c	d	f
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	0
1	1	1	1	0

f: cd \ ab	00	01	11	10
00	0	0	0	0
01	0	1	1	0
11	1	1	0	1
10	0	0	0	0

$f = b\bar{c}d + \bar{b}cd + \bar{a}cd$

$f = d(b \oplus c) + d\bar{a}c$



b) Implement the previous circuit using ONLY 2-to-1 MUXs (AND, OR, NOT, XOR gates are not allowed). (12 pts)

$f(a,b,c,d) = d(b \oplus c) + d\bar{a}c$

$f = \bar{a}f(0,b,c,d) + af(1,b,c,d) = \bar{a}(d(b \oplus c) + dc) + a\big(d(b \oplus c)\big) = \bar{a}g(b,c,d) + ah(b,c,d)$

$g(b,c,d) = \bar{b}(dc) + b(dc + d\bar{c}) = \bar{b}(dc) + b(d)$
$h(b,c,d) = \bar{b}(dc) + b(d\bar{c})$

$t(c,d) = dc = \bar{c}(0) + c(d)$
$u(c,d) = d\bar{c} = \bar{c}(d) + c(0)$